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| EGC_Black | **MATHEMATICS METHODS UNIT 3 & 4**  **TEST 5 2017**  **Calculator Free** |

Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Reading Time: 2 minutes

Total Marks: 15 marks Time Allowed: 15 minutes

**1. [4 marks – 1, 1, 2]**

A survey of 1000 customers to the Teltale help line was conducted in which the time that each customer spent on hold while waiting for help operator. They are shown in 30 second intervals, with the first interval being from 0 to 30 seconds.

Find

a) P(t < 120 seconds)

b) P(60 ≤ t < 150)

c) P(t > 30 | t < 90)

**2. [3 marks -1, 2]**

Complete the sentence.

1. A normally distributed variable X has a mean of 50 and a standard deviation of 7.

Approximately 95% of all scores are located between ………. and …………. .

b) State an estimate for p(X > 57)

**3. [6 marks -3, 3]**

The probability density function f(x) is defined by the piecewise expression

f(x) = {  

1. Determine the value of k.
2. Find P(X < ½)

**4. [2 marks]**

In a Specialist exam, the class achieved an average of 45% with a standard deviation of 15%. The teacher decided to scale the marks so that the mean would be 65% and the standard deviation 12%.

Jason got a raw score of 40%. What would be his scaled score.

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Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Reading Time: 2 minutes

Total Marks: 39 marks Time Allowed: 40 minutes

Question 5 (9 marks)

A bus service departs from a terminus every 30 minutes throughout the day. If a passenger arrives at the terminus at a random time to catch the bus, their waiting time, in minutes, until the next bus departs is a uniformly distributed random variable.

(a) Sketch the graph of the density function of . (2 marks)



(b) What is the probability that a passenger who arrives at the terminus at a random time has to wait no more than 25 minutes for the bus to depart? (1 mark)

(c) Determine . (2 marks)

(d) Determine the value of  for which . (2 marks)

(e) What is the probability that fewer than four passengers, out of a random selection of ten, have to wait at least 25 minutes for the bus to depart?

(2 marks)

**Question 6 (9 marks)**

The heights of 150 Year 12 girls at the local high school are normally distributed with a mean of 162 cm and a standard deviation of 5 cm.

**(a)** Determine:

(i) the probability that a randomly selected girl is less than 164 cm tall. (2 marks)

(ii) the proportion of girls who are between 161 and 163 cm tall, given that

they are less than 164 cm tall. (2 marks)

The shortest ten percent of girls were classified as being too short to be goal shooters in their netball side.

**(b)** What is the height of the shortest girl who would be classified as a goal shooter? (2 marks)

The heights of 120 Year 12 boys at the same school were also normally distributed with a

mean height of 175 cm. 40 of these boys were over 180 cm tall.

**(c)** Find the standard deviation of the heights of the boys. (3 marks)

Question 7 (8 marks)

The random variable denotes the number of hours that a business telephone line is in use per nine hour working day.

The probability density function of is given by ,

where , and are constants.

(a) If and , determine the value of . (2 marks)

(b) Let , and .

(i) The business is open for work for 308 days per year. On how many of these days can the business expect the phone line to be in use for more than eight hours?

(2 marks)

(ii) Determine, correct to two decimal places, the mean and variance of . (4 marks)

Question 8 (6 marks)

The speeds of 250 vehicles, on a section of freeway undergoing roadworks with a speed limit of 60 kmh-1, had a mean and standard deviation of 56.9 kmh-1 and 3.6 kmh-1 respectively. A summary of the data is shown in the table below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Speed ( kmh-1) |  |  |  |  |  |
| Relative frequency | 0.024 | 0.272 | 0.504 | 0.188 | 0.012 |

(a) Use the table of relative frequencies to estimate the probability that the next vehicle to pass the roadworks

(i) was not exceeding the speed limit. (1 mark)

(ii) had a speed of less than 65 kmh-1, given they were exceeding the speed limit.

(1 mark)

(b) Subsequent tests on the measuring equipment discovered that it had been wrongly calibrated. The correct speed of each vehicle, , could be calculated from the measured speed, , by increasing by 6% and then adding 1.7.

(i) Calculate the adjusted mean and standard deviation of the vehicle speeds.

(2 marks)

(ii) Determine the correct proportion of vehicles that were speeding. (2 marks)

Question 9 (7 marks)

A hardware store sells stakes, of nominal length 1.8 metres, to be used for supporting newly planted trees. The length, metres, of the stakes can be modelled by a normal distribution with mean 1.85 and standard deviation .

(a) If , determine

(i) the probability that a randomly chosen stake is shorter than 1.8 metres. (1 mark)

(ii) the probability that a randomly chosen stake is longer than 1.79 m given that it is shorter than 1.8 metres. (2 marks)

(iii) the value of , if the longest 15% of stakes exceed metres in length. (1 mark)

(b) A large number of stakes were measured and it was found that 97% of them were longer than their nominal length. Show how to use this information to deduce that the value of is 0.027 when rounded to three decimal places. (3 marks)